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Comparison of some well-known PID tuning formulas

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Abstract

Criteria based on disturbance rejection and system robustness are proposed to assess the performance of PID controllers. A simple robustness measure is defined and the integral gains of the PID controllers are shown to be a good measure for disturbance rejection. An analysis of some well-known PID tuning formulas reveals that the robustness measure should lie between 3 and 5 to have a good compromise between performance and robustness.

Keywords: PID tuning; Robustness; Structured singular value; Performance

1. Introduction

PID controllers are widely used in industry due to their simplicity and ease of re-tuning on-line (Astrom & Hagglund, 1995). In the past four decades, there are numerous papers dealing with the tuning of PID controllers. A natural question arises: how can the PID settings obtained by different methods be compared? A simple answer is to use step responses of the closed-loop systems and compare the overshoot, rise time and settling time. An alternative is to use the integral error as a performance index. However, these time domain performance measures do not address directly another important factor of a closed-loop system—robustness. It is a well-known fact that models used for controller tuning or design are often inaccurate, so a PID setting based on optimization assuming an accurate model will generally not be guaranteed to be robust. For a fair comparison of different PID settings, both time domain performance and frequency domain robustness should be considered (Shinskey, 1990).

For a single-input-single-output (SISO) process, gain and phase margins are good measures of system robustness. Ho, Gan, Tay, and Ang (1996) and Ho, Hang, and Zhou (1995) compared gain-phase margins of some well-known PID tuning formulas. They show that the load-based tuning methods give gain margins of about 1.5 and phase margins range from 30° to 60°, while the setpoint-based tuning methods give gain margins of about 2 and phase margins of about 65°.

In this paper, we will propose a simple method to analyze system robustness and performance, and then compare some well-known PID tuning formulas and observe some interesting results. It should be noted that the purpose of this paper is to find a simple robustness measure so that we can compare different PID settings. Though we adopt the well-known methods in checking system robustness and time domain performance, our main contribution can be summarized as follows:

(1) We propose to use a simple method to measure the robustness of a system. The measure is more suitable for the purpose of comparing PID controllers than other measures, since it is applicable to multivariable systems and it bounds the sensitivity and the complementary sensitivity functions simultaneously.
(2) By using the measure, we examine the well-known PID tuning formulas and draw some interesting conclusions.

All the symbols used in this paper are common in the area of robust control. The definition of the H_∞ norm and the structured singular value can be referred to Zhou and Doyle (1998), and they can be easily computed with the aid of MATLAB μ toolbox

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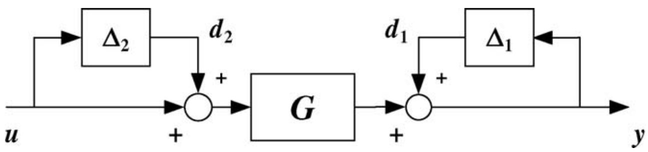

Fig. 1. Uncertainty structure of the model.

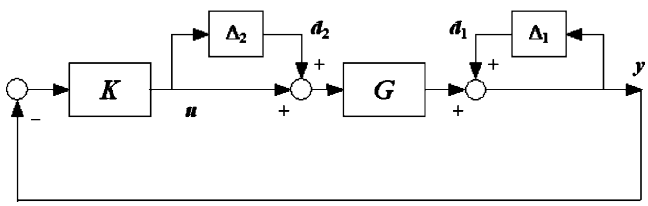

Fig. 2. Uncertainty structure of a unity feedback configuration.

(Balas, Doyle, Glover, Packard, & Smith, 1993). Below is a summary.

Symbols	Description
ω	Frequency
H_∞	The set (Hardy Space) of all stable transfer matrices
$\bar{\sigma}(\cdot)$	The maximum singular value of a matrix
$\underline{\sigma}(\cdot)$	The minimum singular value of a matrix
$\|\|M\|\|_\infty$	The ∞-norm of a transfer matrix M, $\|\|M\|\|_\infty := \sup_\omega \bar{\sigma}(M(\mathrm{j}\omega))$
$\mu_\Delta(M)$	The (complex) structured singular value of M with respect to a (diagonal) block structure $\Delta = \{\mathrm{diag}[\Delta_1, \ldots, \Delta_F]\}$, $\mu_\Delta(M) := 1/\min\{\bar{\sigma}(\Delta) : \det(I - M\Delta) = 0\}$. If M is a transfer matrix, $\mu_\Delta(M) := \sup_\omega \mu_\Delta(M(\mathrm{j}\omega))$

2. Robust analysis

We consider a plant G having the following uncertainty structure (Fig. 1):

$$G_\Delta = (I - \Delta_1)^{-1} G (I + \Delta_2), \text{ with } \Delta_1, \Delta_2 \in H_\infty. \tag{1}$$

It represents simultaneous input multiplicative and inverse output multiplicative uncertainty. Suppose a normalized left coprime factorization of G is $G = \tilde{M}^{-1}\tilde{N}$, then

$$\begin{aligned} G_\Delta &= (\tilde{M} - \tilde{M}\Delta_1)^{-1}(\tilde{N} + \tilde{N}\Delta_2) \\ &=: (\tilde{M} + \Delta_M)^{-1}(\tilde{N} + \Delta_N). \end{aligned} \tag{2}$$

So compared with a coprime factor uncertainty (McFarlane & Glover, 1990), additional structure information ($\Delta_M = -\tilde{M}\Delta_1$ and $\Delta_N = \tilde{N}\Delta_2$) is contained in the uncertainty expression, thus the conservatism of the robust analysis can be reduced.

A unity feedback control system with the uncertain plant G_Δ is shown in Fig. 2.

To analyze the robust stability of the closed-loop system, we transfer the uncertain system to an M-Δ structure shown in Fig. 3. Here, M is the transfer matrix from signals d_1, d_2 to signals y, u and given by

$$M := \begin{bmatrix} (I + GK)^{-1} & (I + GK)^{-1} \\ -K(I + GK)^{-1} & -K(I + GK)^{-1}G \end{bmatrix}. \tag{3}$$

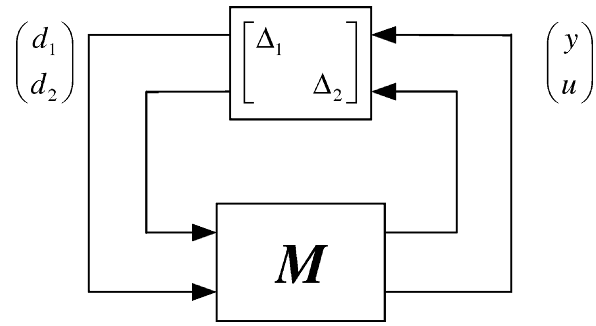

Fig. 3. M-Δ configuration.

Define

$$\Delta := \begin{bmatrix} \Delta_1 & 0 \\ 0 & \Delta_2 \end{bmatrix}. \tag{4}$$

By the small μ theorem (Zhou & Doyle, 1998), the closed-loop system in Fig. 2 is robustly stable for all $||\Delta||_\infty \le \gamma$ if and only if

$$\varepsilon := \mu_\Delta(M) < 1/\gamma. \tag{5}$$

Thus, $\mu_\Delta(M)$ is a measure of system robustness.

In Panagopoulos and Astrom (2000) and Panagopoulos, Astrom, and Hagglund (2002), the following robustness criterion was proposed,

$$\gamma := \left\| \begin{bmatrix} G \\ I \end{bmatrix} (I + GK)^{-1} [-K \quad I] \right\|_\infty. \tag{6}$$

For a single loop control system, it is easy to show that

$$\gamma = \left\| \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} M^{\mathrm{T}} \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \right\|_\infty = ||M^{\mathrm{T}}||_\infty = ||M||_\infty. \tag{7}$$

where M is given by (3), and M^{T} denotes the transpose of M.

We note that sometimes γ is not suited for robustness measure. For example, consider a high-order plant having a large roll-off rate at high frequencies

$$P = \frac{1}{(s+1)^5}, \tag{8}$$

and a practical PID controller tuned by the Ziegler-Nichols method (Ziegler & Nichols, 1942)

$$K = 1.7313 \left(1 + \frac{1}{4.3240s} + \frac{1.0810s}{1.0810/10s + 1} \right). \tag{9}$$

Fig. 4 shows $\mu_\Delta(M(\mathrm{j}\omega))$ and $\bar{\sigma}(M(\mathrm{j}\omega))$ of the unity feedback control system. It can be read from the figure that $\gamma = 19.0$ and $\varepsilon = 3.7$. The peak of $\bar{\sigma}(M(\mathrm{j}\omega))$ at the high frequencies gives wrong indication of the robustness of the closed-loop system, since it can be easily reduced with little performance degradation by incorporating a low-pass filter in the controller. The peak at the mid-frequency makes more sense.

So the additional structure information ($\Delta_M = -\tilde{M}\Delta_1$ and $\Delta_N = \tilde{N}\Delta_2$) used in the proposed method leads the robustness

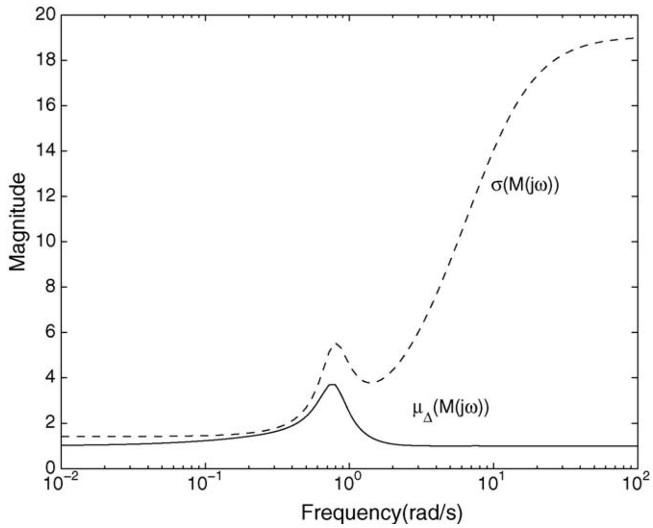

Fig. 4. Structured singular values vs. maximum singular values.

measure to a reasonable range and thus a fair comparison can be made for different controllers.

We should notice that this measure is just a rough measure of system robustness, since we have ignored the detailed structure of the uncertainty and chosen a special structure. For a detailed robust stability analysis of a closed-loop system, μ-analysis should be used. However, since the μ-analysis procedure involves finding the detailed uncertainty structure of the plant model, it is very complex and problem-specific. We note that most of the PID tuning methods do not incorporate model uncertainty, so it is unfair to compare them under a specified model uncertainty structure. The robustness measure we proposed assumes a simple and general uncertainty structure, so the measure gives a simple and somewhat fair comparison of the PID controllers.

3. Performance analysis

The integral error is generally accepted as a good measure for system performance. The followings are some commonly used criteria based on the integral error for a step setpoint or disturbance response:

$$\mathrm{IAE} = \int_0^\infty |\mathrm{e}(t)|\mathrm{d}t, \quad \mathrm{ISE} = \int_0^\infty \mathrm{e}(t)^2\,\mathrm{d}t,$$
$$\mathrm{ITAE} = \int_0^\infty t|\mathrm{e}(t)|\mathrm{d}t, \quad \mathrm{ITSE} = \int_0^\infty t\mathrm{e}(t)^2\,\mathrm{d}t, \tag{10}$$
$$\mathrm{ISTE} = \int_0^\infty t^2\mathrm{e}(t)^2\,\mathrm{d}t.$$

A more convenient criterion is the integral of the error (IE).

$$\mathrm{IE} = \int_0^\infty \mathrm{e}(t)\,\mathrm{d}t. \tag{11}$$

Clearly, if the response is critically damped, IE will be equal to IAE. However, if it is weakly damped, then IE will not be suitable as a performance measure. Nevertheless, as pointed out

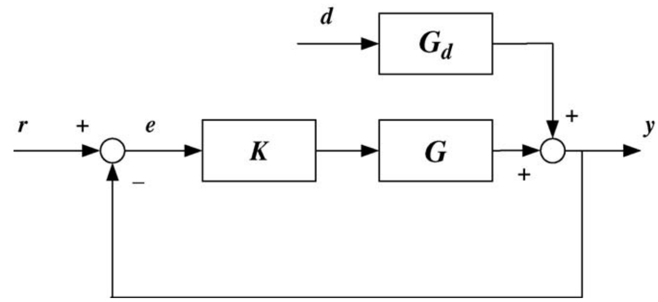

Fig. 5. Typical unity feedback configuration.

in Astrom and Hagglund (1995) for a single-loop process,

$$\int_0^\infty \mathrm{e}(t)\,\mathrm{d}t = \frac{1}{K_\mathrm{i}}, \tag{12}$$

where K_i is the integral gain of the controller. So it is easy to compute. Further, as we will discuss below, a larger integral gain usually implies less robustness. Thus, combined with the robustness measure, IE, or the integral gain, can be a good measure of system performance.

An advantage of using the integral gain is that it can be easily extended to a multiloop system. Consider the unity feedback system (possibly multiloop) shown in Fig. 5.

The transfer function from d to y is

$$T_{yd} = (I + GK)^{-1}G_d. \tag{13}$$

Assume that controller K has integral action, we can decompose it as

$$K(s) = \frac{K_\mathrm{i}}{s} + K_m(s). \tag{14}$$

where K_i is the integral gain and K_m is the part of the controller without integral action. Then at the low-frequency region (where ω is small), we have

$$\begin{aligned}\bar\sigma(T_{yd}(\mathrm{j}\omega)) &= \bar\sigma((I + G(\mathrm{j}\omega)K(\mathrm{j}\omega)^{-1}G_d(\mathrm{j}\omega)) \\ &= \bar\sigma((I + G(\mathrm{j}\omega)K_m(\mathrm{j}\omega) + G(\mathrm{j}\omega)K_\mathrm{i}/(\mathrm{j}\omega))^{-1}G_d(\mathrm{j}\omega))^{-1} \\ &\approx \bar\sigma(G(\mathrm{j}\omega)K_\mathrm{i}/(\mathrm{j}\omega))^{-1}G_d(\mathrm{j}\omega)).\end{aligned} \tag{15}$$

For any compatible matrices A and B, it is well-known that $\bar\sigma(AB) \le \bar\sigma(A)\bar\sigma(B)$ and $\bar\sigma(A^{-1}) = \underline{\sigma}(A)$. So, we have

$$\bar\sigma(T_{yd}) \le |\mathrm{j}\omega|\frac{\bar\sigma(G_d(\mathrm{j}\omega))}{\underline{\sigma}(G(\mathrm{j}\omega))\underline{\sigma}(K_\mathrm{i})}. \tag{16}$$

We note that $\bar\sigma(T_{yd})$ is a measure of the system ability to reject disturbance. In industrial processes, the disturbance usually occurs at low frequency. So the above inequality shows that to reject a disturbance the most important element of a controller is its integral gain, or specifically, the minimum singular value of the integral gain. Thus, the minimum singular value of the integral gain is a natural extension as a performance measure for multivariable processes.

In summary, we can assess the performance of a controller by evaluating the minimum singular value of its integral gain, and assess the robustness by the robustness measure ε defined in (5). We note that the performance criterion mainly concerns the disturbance response. The setpoint response can always be improved by using a setpoint filter or setpoint weighting. Since no specific model is required to derive the robustness and performance criteria, the proposed method can be applied to a variety of plant models, no matter they are stable, integrating or unstable; single-loop or multiloop.

4. Comparison of tuning formulas

Examples can be given to evaluate various PID design or tuning methods found in the literature for a certain process via the proposed criteria. However, since a specific method might be effective for a specific plant model, it is hazardous to draw general conclusions on which method is the best (in fact no best at all). What we can conclude is that some methods show better performance in disturbance rejection and/or robustness than other methods.

In this section, we will apply the criteria proposed in the previous section to analyze several PID tuning techniques found in the literature. We restrict our comparison under the following conditions:

(1) The process model is first-order with deadtime (FOPDT)

$$P(s) = \frac{k}{Ts+1}\,\mathrm{e}^{-\tau s}. \tag{17}$$

(2) The following PID tuning formulas are considered:
   (a) Ziegler–Nichols (Z–N) method (Ziegler & Nichols, 1942). There are two versions of Z–N method. One depends on the reaction curve, and the other, the ultimate gain K_u and the ultimate period T_u. Here, we consider the latter.
   (b) Cohen–Coon (C–C) method (Cohen & Coon, 1953). The C–C method is based on reaction curve. A model with one tangent and point is derived first to tune the PID controller. For FOPDT model, the PID parameters can be directly related to model parameters.
   (c) Internal model control (IMC) method. There are several versions of IMC-PID methods. We consider the one first proposed in Rivera, Morari, and Skogestad (1986). It has a tuning parameter. The smaller it is, the better performance the closed-loop system will have, and the less robust the closed-loop system is. Here, the tuning parameter λ is chosen as 0.25 of the delay, the smallest value as suggested in the reference.
   (d) Gain-phase margin (G-P) method. The G-P method was proposed in Astrom and Hagglund (1984) and further discussed in Ho, Hang, and Cao (1995). Since different pair of gain-phase margin will result in different PID settings, here we choose the tuning formula given in Zhuang and Atherton (1993) where the gain-phase margin is optimized.
   (e) Optimum integral error for load disturbance (IAE-load, ITAE-load, ISE-load, ISTE-load), and for setpoint change (IAE-setpoint, ITAE-setpoint, ISE-setpoint, ISTE-setpoint) methods. There are many versions of the integral-error based methods. The original references can be found in papers and books written in the 1960's. Here, IAE, ITAE optimal tunings are adopted from Ho et al. (1996), and ISE, ISTE optimal tunings can be found in Zhuang and Atherton (1993).

The tuning formulas for the methods considered are shown in Table 1. Some comments on the model and the tuning formulas considered are:

(1) The FOPDT model is widely used in process control, and there are lots of tuning methods based on the model. An important factor of the model is the normalized delay (ratio of the time delay and the time constant τ/T), which shows how large the 'real' delay of the system is Astrom, Hang, Persson, and Ho (1992). We will discuss FOPDT models whose τ/T is less than 1, for the reason that if τ/T is greater than 1, the process will be regarded as having a large time delay and PID control structure is not recommended for such processes. To achieve good performance, deadtime compensator structure, such as a Smith predictor must be used.
(2) The tuning formulas considered seem quite out of date. However, we believe that these methods are representative, and they are generally bases of comparison for more recent tuning methods. We note that there are lots of PID design and tuning methods reported recently in the literature, for example, Grassi et al. (2001), Ho, Lim, and Xu (1998), Kristiansson and Lennartson (2002), the performance of these methods can be evaluated via the proposed method. Due to space limit and our emphasis, we do not include them here.
(3) While there are many versions of integral-error based, IMC-based and gain-phase based formulas, we only consider one of them. Other versions should be essentially the same.
(4) The formulas use different PID controller structures. We transform the series form to the parallel form, and compare them based on the following practical PID structure:

$$K = K_\mathrm{p}\left(1 + \frac{1}{T_\mathrm{i}s} + \frac{T_d s}{T_d/10s+1}\right). \tag{18}$$

Fig. 6 shows the robustness measures ε for the PID controllers tuned by the listed methods against the normalized delay, and Fig. 7 shows the corresponding integral gains.

The following can be observed:

(1) IMC and G-P methods can be regarded as setpoint-based tuning methods, since the integral gains of the PID controllers tuned by the two methods are close to those of the setpoint-based optimum integral error methods. Similarly, Z–N and C–C methods can be regarded as load-based tuning methods.
(2) The robustness measures of the setpoint-based methods are less than 4 except the ITAE-setpoint method for processes with large delay; while those for the load-based methods are

Table 1
PID tuning formulas

Controller	$K'_c\left(1+\frac{1}{T'_i s}\right)\left(\frac{T'_d s+1}{0.1T'_d s+1}\right)$		
	K'_c	T'_i	T'_d
IAE-set-point	$\frac{0.65}{k}\left(\frac{\tau}{T}\right)^{-1.04432}$	$\frac{T}{0.9895+0.09539\tau/T}$	$0.50814T\left(\frac{\tau}{T}\right)^{1.08433}$
IAE-load	$\frac{0.980890}{k}\left(\frac{\tau}{T}\right)^{-0.76167}$	$\frac{T}{0.91032}\left(\frac{\tau}{T}\right)^{1.05211}$	$0.59974T\left(\frac{\tau}{T}\right)^{0.89819}$
ITAE-setpoint	$\frac{1.12762}{k}\left(\frac{\tau}{T}\right)^{-0.80368}$	$\frac{T}{0.99783+0.02860\tau/T}$	$0.42844T\left(\frac{\tau}{T}\right)^{1.0081}$
ITAE-load	$\frac{0.77902}{k}\left(\frac{\tau}{T}\right)^{-1.06401}$	$\frac{T}{1.14311}\left(\frac{\tau}{T}\right)^{0.70949}$	$0.57137T\left(\frac{\tau}{T}\right)^{1.03826}$
Controller	$K_p\left(1+\frac{1}{T_i s}+T_d s\right)$		
	K_p	T_i	T_d
ISE-setpoint	$\frac{1.048}{k}\left(\frac{\tau}{T}\right)^{-0.897}$	$\frac{T}{1.195-0.368\tau/T}$	$0.489T\left(\frac{\tau}{T}\right)^{0.888}$
ISE-load	$\frac{1.473}{k}\left(\frac{\tau}{T}\right)^{-0.970}$	$\frac{T}{1.115}\left(\frac{\tau}{T}\right)^{-0.753}$	$0.550T\left(\frac{\tau}{T}\right)^{0.948}$
ISTE-setpoint	$\frac{1.042}{k}\left(\frac{\tau}{T}\right)^{-0.897}$	$\frac{T}{0.987-0.238\tau/T}$	$0.385T\left(\frac{\tau}{T}\right)^{0.906}$
ISTE-load	$\frac{1.468}{k}\left(\frac{\tau}{T}\right)^{-0.970}$	$\frac{T}{0.942}\left(\frac{\tau}{T}\right)^{-0.725}$	$0.443T\left(\frac{\tau}{T}\right)^{0.939}$
Z–N	$0.6K_u$	$0.5T_u$	$0.125T_u$
C–C	$\frac{T}{k\tau}\left(\frac{16T+3\tau}{12T}\right)$	$\frac{\tau(32+6\tau/T)}{13+8\tau/T}$	$\frac{4\tau}{11+2\tau/T}$
IMC[a]	$\frac{2T+\tau}{2k(\lambda+\tau)}$	$T+\frac{\tau}{2}$	$\frac{T\tau}{2T+\tau}$
G-P[b]	$mK_u\cos\phi$	αT_d	$\frac{\tan\phi+\sqrt{4/\alpha+\tan^2\phi}}{2\omega_c}$

[a] $\lambda \geq 0.25\tau$ as suggested in Rivera et al. (1986).
[b] $\phi = 33.8^\circ\,(1-0.97\,e^{-0.45kK_u})$, $m = 0.614(1-0.233\,e^{-0.347kK_u})$, $\alpha = 0.413(3.302kK_u+1)$. K_u is the ultimate gain and ω_c is the ultimate frequency.

greater than 4, except the Z–N method for processes with small delay and the ITAE-load method for processes with large delay. So the PID controllers tuned by the setpoint-based methods are usually more robust than those tuned by the load-based methods.

(3) The PID controllers tuned by the setpoint-based methods for processes with small delay have very small integral gains and thus have sluggish load responses. On the other hand, the PID controllers tuned by the load-based methods have very large integral gains but are generally not robust.

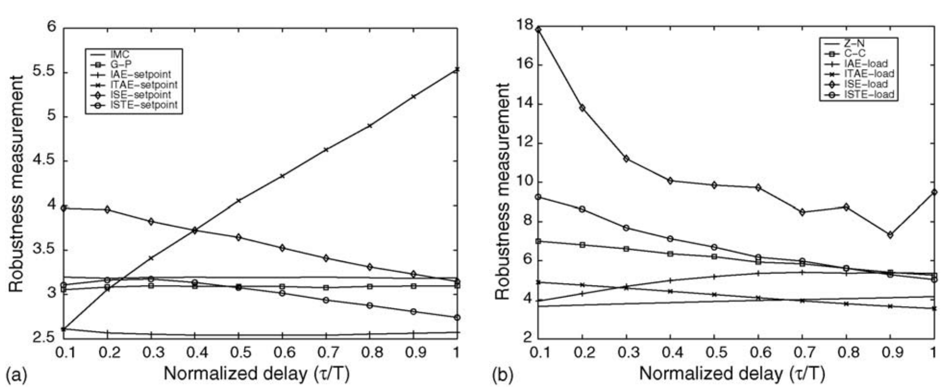

Fig. 6. Robustness measures: (a) setpoint-based methods and (b) load-based methods.

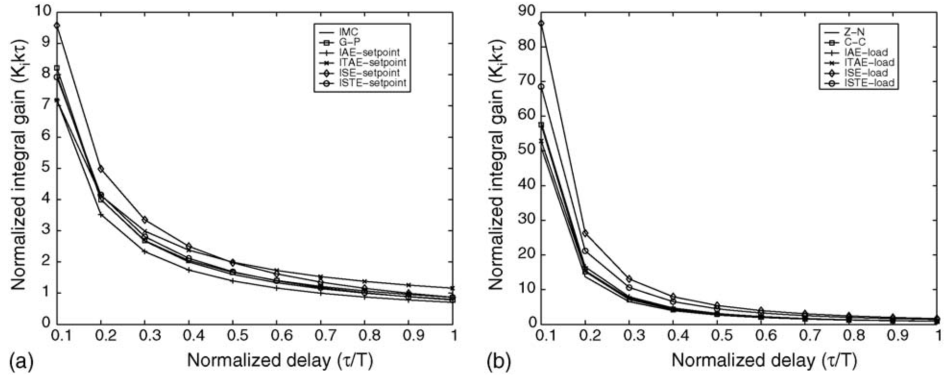

Fig. 7. Integral gains: (a) setpoint-based methods and (b) load-based methods.

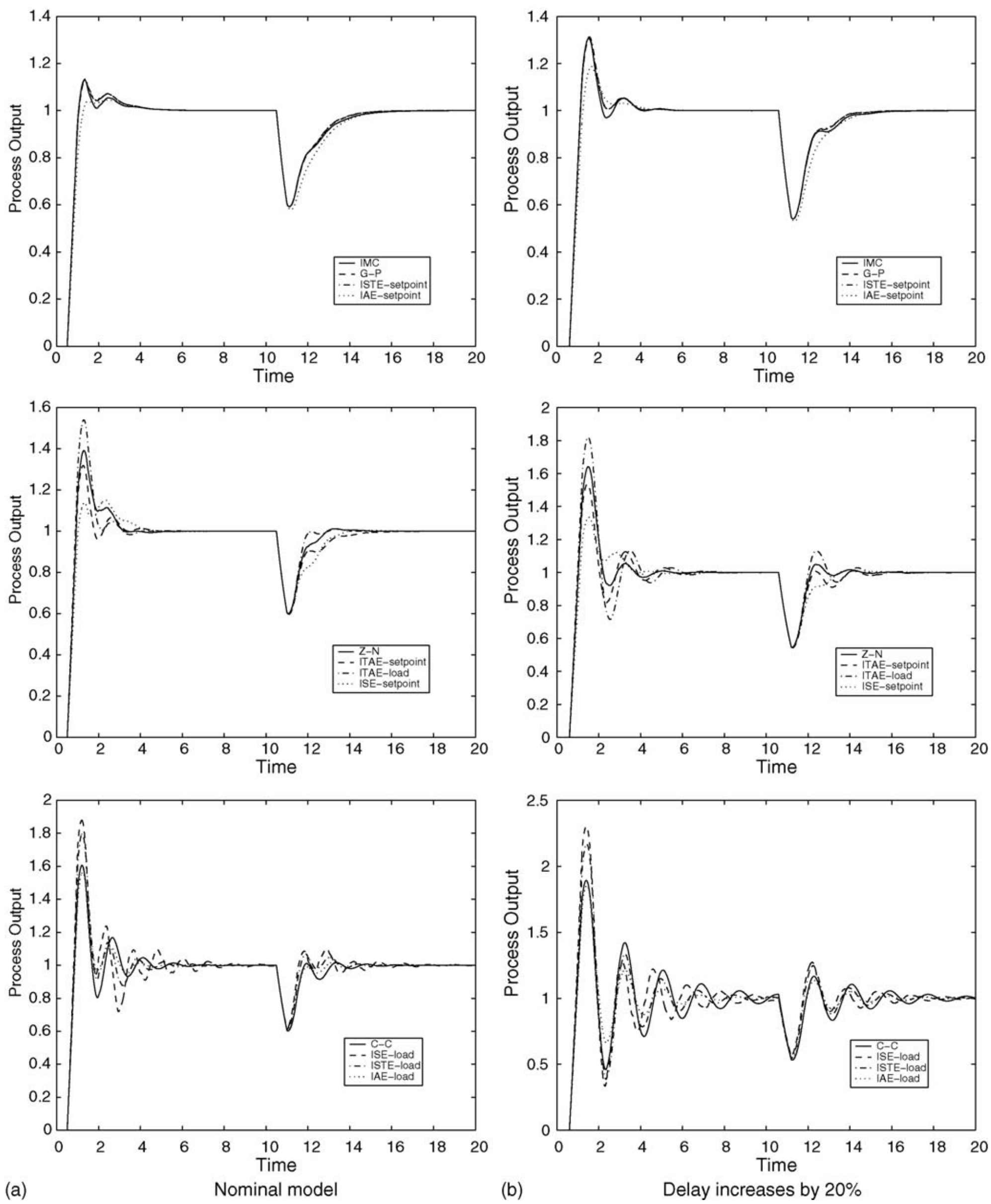

Fig. 8. Responses of different PID settings: (a) nominal model and (b) delay increases by 20%.

(4) All of these methods give similar integral gains for processes with large delay, but the setpoint-based methods (except ITAE-setpoint) have smaller robustness measures, so they are better than the load-based methods for processes with large delay.
(5) Among the load-based methods, only Z–N and ITAE-load methods are acceptable. ISE-load, ISTE-load, IAE-load and C–C methods should not be used since the PID controllers tuned by them are too aggressive. The robustness of the PID controllers tuned by the Z–N method become worse as the delay becomes larger, so it should only be used for processes with small delay.
(6) Among the setpoint-based methods, IMC and G-P methods have almost constant robustness measures, but the G-P method has smaller robustness measure and larger integral gain, so it is slightly better than the IMC method. The IAE-setpoint method has the smallest integral gains and the best robustness measures thus it is too conservative. The ISE-setpoint method has larger integral than IMC and G-P methods with a sacrifice on robustness, and the ISTE-setpoint method has smaller integral than IMC and G-P methods with slightly larger robustness. Like the Z–N method, the robustness of the PID controllers tuned by the ITAE-setpoint method become worse as the delay becomes larger, so it should only be used for processes with small delay, too.
(7) Extensive simulations show that when the robustness measure ε is larger than 5, then the closed-loop system will not have sufficiently large robust margin. On the other hand, if it is less than 3, the integral gain will not be sufficiently large. The best compromise for the robustness measure is between 3 and 5.

To illustrate the above observation, consider the following process:

$$P(s) = \frac{1}{s+1}\,\mathrm{e}^{-0.5s} \tag{19}$$

Table 2 shows the PID setting tuned by the listed methods.

It is clear that the resulting controllers can be divided into three groups:

Table 2
PID settings tuned by various methods

	K_p	T_i	T_d	K_i	ε
IMC	2	1.25	0.2	1.6	3.193
G-P	1.938	1.164	0.208	1.665	3.092
ISE-setpoint	1.952	0.989	0.264	1.973	3.646
ISTE-setpoint	1.940	1.152	0.206	1.684	3.078
IAE-setpoint	1.674	1.204	0.192	1.390	2.544
ITAE-setpoint	2.393	1.201	0.175	1.992	4.054
Z–N	2.285	0.855	0.214	2.617	3.912
C–C	2.917	1.209	0.167	2.833	6.024
ISE-load	2.885	0.532	0.285	5.422	9.852
ISTE-load	2.876	0.642	0.231	4.477	6.698
IAE-load	2.673	0.852	0.20	3.139	5.188
ITAE-load	2.475	0.813	0.183	3.045	4.258

(1) Controllers tuned by IMC, G-P, ISTE-setpoint and IAE-setpoint methods have small integral gains and small robustness measures.
(2) Controllers tuned by Z–N, ITAE-setpoint, ITAE-load and ISE-setpoint methods have medium integral gains and medium robustness measures.
(3) Controllers tuned by C–C, ISE-load, ISTE-load and IAE-load methods have large integral gains and large robustness measures.

Fig. 8 shows the closed-loop system responses of all the PID controllers for a step setpoint change of magnitude 1 at $t=0$ following a step load disturbance of magnitude 1 at $t=10$ for the nominal model and for the perturbed case that the delay increases by 20%. It is observed that the integral gains and robustness measures given by the first group are too small, thus the closed-loop systems are very robust but the load rejection performance can be further improved. The PID settings in this group will be referred as *conservative*. The integral gains and robustness measures given by the third group are too large, thus the closed-loop systems show oscillatory responses and are not robust. The PID settings in this group will be referred as *aggressive*. The second group gives *proper* integral gains and robustness measures. The Z–N method has the best compromise between robustness and performance for the process considered.

5. Conclusions

Criteria based on disturbance rejection and system robustness were proposed to assess the performance of PID controllers. Several well-known PID tuning formulas were analyzed and it was observed that the robustness measure should lie between 3 and 5 to have a good compromise between performance and robustness.

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